A STUDY ON MAXIMUM RESPONSES ON ELASTO-PLASTIC STRUCTURE EXCITED BY SYNTHESIZED GROUND MOTIONS

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ABSTRACT: A number of artificial earthquake ground motions compatible with time-frequency characteristics of recorded actual earthquake ground motions as well as the given target response spectrum are generated using wavelet transform. The coefficient of variation (C. O. V.) of maximum displacement on elasto-plastic SDOF systems excited by these artificial ground motions are numerically evaluated.

Key Words: Seismic motion, earthquake resistant, nonlinear vibration, wavelet, variation of responses

INTRODUCTION

For a design basis earthquake ground motion like a Level-1 or a Level-2 design earthquake ground motion, an artificial earthquake ground motion is usually defined to be compatible with a given design response spectrum. It has been known that the maximum values of elasto-plastic responses of a structure subjected to a group of artificial earthquake ground motions generated to be compatible with the same design response spectrum change in no small way (Kitahara 2001).

For the measure of earthquake ground motion closely related to the maximum value of elasto-plastic response of a structure, total input energy, input energy rate of its time differential and instant input energy of its increment value per second, etc. based on the energy theory are recently taken up and relationship between those factors and maximum ductility factor is investigated (Kato 1975, Ohi 1991, Kuwamura 1997, Nakamura 1998). The input energy to a structure from an earthquake ground motion

shows nonstationary time history due to its nonstationary characteristics and nonlinearity of structure, and it is known that the structural damage rate depends on this nonstationarity. In order to set more rational design basis earthquake ground motion, it is important that the nonstationary characteristics of a real earthquake ground motion which could hit a target structure in the future must be reflected in the design basis earthquake ground motion and its influence on the nonlinear response of the structure is taken account of in the design of the structure.

The method using Fourier phase characteristic (Kimura 1986) and the method using wavelet transform (Maeda 2002) are proposed as for synthesis of an artificial earthquake ground motion considering nonstationary characteristics of it. Masuda et al. proposed the method generating the artificial earthquake ground motion compatible with the given response spectrum and having nonstationary time-frequency characteristics by using wavelet transform of velocity response function (Masuda 2002a, Masuda 2002b).

In this paper, from the viewpoint of generating a rational design basis earthquake ground motion, the influence of frequency nonstationary characteristic of earthquake ground motion on the elasto-plastic response of structure is examined using a group of artificial earthquake ground motions compatible with the same response spectrum and having the time-frequency characteristics of the recorded earthquake's ground motions by Masuda's method, and the examples of the variance of maximum displacement responses of elasto-plastic structure are shown.

GENERATION OF ARTIFICIAL EARTHQUAKW GROUND MOTION

The relative velocity response $\dot{x}(t)$ of a single degree of freedom (SDOF) structure with a natural period *T* and damping ratio ζ subjected to earthquake ground acceleration f(t) is given by Eq.(1) using the velocity impulse response function.

$$\dot{x}(t) = \int_{-\infty}^{\infty} f(t) g_V^{T,\varsigma}(t-\tau) d\tau$$
⁽¹⁾

where, $g_V^{T,\varsigma}(t)$ is the velocity impulse response function as

$$g_{V}^{T,\varsigma}(t) = \begin{cases} -e^{-\frac{2\pi\varsigma}{T}t} (\cos\frac{2\pi\sqrt{1-\varsigma^{2}}}{T}t - \frac{\varsigma}{\sqrt{1-\varsigma^{2}}} \sin\frac{2\pi\sqrt{1-\varsigma^{2}}}{T}t) & t \ge 0\\ 0 & t < 0 \end{cases}$$
(2)

The velocity response spectrum $S_V(T, \varsigma)$ is defined as the maximum amplitude of velocity response in terms of T and ς , and $S_V(T, \varsigma)$ is given by

$$S_{V}(T,\varsigma) = \max_{t} \left| \dot{x}(t) \right| = \max_{t} \left| \int_{-\infty}^{\infty} f(t) g_{V}^{T,\varsigma}(t-\tau) d\tau \right|$$
(3)

Introducing a function $\psi(t)$ defined as

$$\psi(t) = g_V^{1,\varsigma}(-t) \tag{4}$$

Using this function $\psi(t)$, the impulse velocity response $g_V^{T,\zeta}(t)$ of a structure with natural period T is given by

$$g_V^{T,\varsigma}(t) = \psi(\frac{-t}{T}) \tag{5}$$

Substituting this equation into Eq.(3), the velocity response spectrum $S_V(T, \varsigma)$ is described as Eq.(6) using a function $\psi(t)$.

$$S_{V}(T,\varsigma) = \max_{t} \left| \int_{-\infty}^{\infty} \psi(\frac{t-\tau}{T}) f(t) dt \right|$$
(6)

where the integral in the right hand side forms a continuous wavelet transform, in which acts as an analyzing wavelet $\psi(t)$ with the scale parameter T and the sift parameter τ . velocity response spectrum can be described in terms of wavelets, i.e., it can be interpreted as the maximum amplitude of the wavelet transform of the earthquake ground motion at each scale:

$$S_{V}(T,\varsigma) = \max_{t} \left| W_{\psi} \{f\}(T,\tau) \right| \tag{7}$$

where $W_{\psi}{f}(T,\tau)$ denotes the wavelet transform of the earthquake ground acceleration f(t). Therefore, using this relation, we can compose the ground acceleration compatible to the given design response spectrum and the given time-frequency characteristic through inverse wavelet transform.

In this study, the Osaki's spectrum (Osaki 1995) for the magnitude of 7 and the epicenter distance of 20 km of earthquake, shown in Fig.1, is adopted as the design spectrum.



5 time-frequency functions derived actual recorded ground motions which recently caused extensive damage for structures or maximum ground acceleration more than 1 m/s^2 are adopted as the target time-frequency characteristics to synthesize the artificial ground motions. The list of selected actual ground motions is Table 1 and the wave forms of selected ground motions are shown in Fig. 2. The durations of these ground motions are taken shorter time than original record.

No.	Earthquake name	Origin time	Location of	Direction	Maximum
			observatory		acceleration
1	The 2011 off the Pacific	2011/03/11/14:46	Tsukidate(MYG004)	NS	27.0 m/s^2
2	coast of Tohoku Earthquake		Shiogama(MYG012)	EW	19.7
3			Hitachi(IBR003)	NS	16.0
4			Hokota(IBR013)	NS	13.5
5	The Niigataken Chuetsu-oki	2007/0716/10:13	Kashiwazaki	EW	5.14
	Earthquake in 2007		(NIG018)		

Table 1 List of selected actual earthquake's ground motions

The 30 artificial ground motions are synthesized for each time-frequency characteristic. The examples of synthesized ground motions and the examples of velocity response spectra of synthesized ground motions are shown in Fig. 3 and Fig. 4 respectively. From Fig. 2 and Fig. 3, No. 1 and No.2 earthquake ground motions and synthesized ground motions have dual shocks and others have single shock. The synthesized ground motions are fairly similar to original earthquake ground motions. From Fig. 4 we can see that the velocity response spectra of synthesized ground motions compatible with the target spectrum. The mean value of the coefficients of variation (COV) of the velocity response spectra of the synthesized ground motions are $0.048 \sim 0.057$.







Fig. 3 Examples of synthesized ground motions



ground motions

The time-frequency characteristics derived from 5 actual earthquake ground motions (Table 1) are shown in Fig. $5 \sim$ Fig. 9.





VARIANCE OF MAXIMUM DISPLACEMENT RESPONSES ON BILINEAR SYSTEM

The single degree of freedom system (SDOFS) which is made of a mass, bi-linear hysteretic spring element and viscous damping element is assumed. The equation of motion for this SDOFS is expressed as Eq. (8).

$$m\ddot{x} + c\dot{x} + F(x, \dot{x}) = -m\ddot{x}_{g} \tag{8}$$

Where, *m*, *c*, $F(x,\dot{x})$, *x* and \ddot{x}_g denote mass of SDOFS, damping coefficient, bi-linear restoring force, relative displacement of mass and acceleration applied to SDOFS respectively. The equation of motion is numerically solved by linear acceleration method. On the bi-linear restoring force model, the ratio of the post-yield stiffness k_p to the initial stiffness k_e before yield is assumed 1/100. And yield force levels F_y are assumed as relative input intensities α in Eq. (9) are $1.0 \sim 2.0$.

$$\alpha = \frac{E(\max\left|\ddot{x}_g\right|)m}{F_y} \tag{9}$$

Where $E(\cdot)$ denotes the mean value. The natural period T_0 in linear range shown as Eq. (10)

$$T_0 = 2\pi \sqrt{\frac{m}{k_e}} \tag{10}$$

and the damping ratio ς of the viscous damping expressed as Eq. (11)

$$\varsigma = \frac{c}{2\sqrt{mk_e}} \tag{11}$$

are taken as $T_0=0.05 \sim 2.0$ s and $\varsigma = 0.02$ respectively.

The spectra of maximum displacement responses $\max |x|$ of the mass for the natural period T_0 of 0.05 s ~2.0 s to the synthesized ground motions with relative intensity α of 2.0 and each type of time-frequency characteristic are shown in Fig. 10 ~ Fig. 14 . In these figure, the COV of the maximum displacement responses are also shown. From these figures, the COV vary with the period and the type of time-frequency characteristic.



Fig. 10 Max. displacement response by No. 1 synthesized ground motion



Fig. 12 Max. displacement response by No. 3 synthesized ground motion



Fig. 14 Max. displacement response by No. 5 synthesized ground motion

 10° Max. displacement response (m) COV (dimensionless value) 10 no 2 earthq t-f alpha=2.0 10 nax. displacement 10 mean value standard deviation 10 10^{-2} 10-1 10 10^{1} Period (s)

Fig. 11 Max. displacement response by No. 2 synthesized ground motion



Fig. 13 max. displacement response by No. 4 Synthesized ground motion

The COV at α of 0.1, 1.0 and 2.0 and at T_0 of 0.1 s, 0.2 s, 0.5 s and 1.0 s for each type of time-frequency characteristic of the synthesized ground motions are shown in Table 1. The response at α of 0.1 means linear response. And the spectra of COV at α of 1.0 and 2.0 for each type of time-frequency characteristic 0f the synthesized ground motions are shown in Fig. 15. It is recognized that COV becomes large as the value of α increase and varies with T_0 and the type of time-frequency characteristic from these Table 1 and Fig. 15.

Relative	Period	Type of synthesized ground motions					
input	$T_0(s)$	Time-frequency	Time-frequency	Time-frequency	Time-frequency	Time-frequency	
intensity		of No. 1 earthq.	of No. 2 earthq.	of No. 3 earthq.	of No. 4 earthq.	of No. 5 earthq.	
α							
0.1 (*)	0.2	0.0988	0.0963	0.1286	0.1149	0.1046	
	0.5	0.1111	0.0832	0.1060	0.1356	0.1051	
	1.0	0.1777	0.1055	0.1831	0.1508	0.1433	
1.0	0.2	0.2307	0.1328	0.3019	0.2434	0.3495	
	0.5	0.1581	0.1355	0.1785	0.2241	0.2360	
	1.0	0.1785	0.1055	0.1766	0.1925	0.1539	
2.0	0.2	0.3192	0.2962	0.3666	0.2526	0.2529	
	0.5	0.2824	0.2293	0.2573	0.3424	0.3101	
	1.0	0.2091	0.1616	0.2239	0.2834	0.2655	

Table 1 Coefficient of variation of the max. displacement response

(*) linear response



Fig. 15 Coefficient of variation of maximum displacement responses by No.1 ~ No.5 synthesized ground motions

The significant differences between the variances of maximum displacement due to the synthesized ground motions with different type of time-frequency characteristics are studied. In Fig. 16, the ratio of the unbiased variance of the maximum displacement responses due to the ground motions with No. 1 earthquake's time-frequency characteristic to that with No. 2 earthquake's time-frequency characteristic, the ratio of that with No. 1 earthquake's time-frequency characteristic to that with No.3 earthquake's characteristic and the ratio of that with No.1 earthquake's time-frequency characteristic to that with No. 5 earthquake's time-frequency characteristic are shown. The values of F-distribution corresponding to significance level of 5 % are 2.10 and 0.47 drawn by dotted lines. These ratios in this Fig. 16 are inside of the dotted lines at most of the periods, that is to say, there is not significant difference between the numerator and the denominator of the ratio of the variance at most of the periods. The ratio of the unbiased variance of the maximum displacement responses due to the ground motions with No. 3 earthquake's time-frequency characteristic to that with No. 4 earthquake's time-frequency characteristic, the ratio of that with No. 2 earthquake's time-frequency characteristic to that with No. 4 earthquake's time-frequency characteristic and the ratio of that with No. 3 earthquake's time-frequency characteristic to that with No. 5 earthquake's time-frequency characteristic are shown in Fig. 17. The former two ratios are not inside of the dotted lines at most of the periods. The remainder, the ratio of that with No.3 earthquake to that with No. 5earthquake, are inside of the dotted lines at most of the periods, that is to say, there are significant differences between variance of the maximum displacement responses due to the ground motions with No. 3 earthquake's time-frequency characteristic and that with No.4 earthquake's one and between variance of that with No.2 earthquake's one to that with No.4 earthquake's one at most of the periods. However, there is not significant difference between that with No.3 earthquake's one and that with No.5 earthquake's one at most of the periods.



From above descriptions, it is recognized that COV of maximum displacement responses varies with T_0 and the type of the time-frequency characteristic and COV becomes large as α increases. There is not significant difference between COV of that with the time-frequency characteristics of the recorded ground motion at KASHIWAZAKI in The Niigataken Chuetsu-oki Earthquake in 2007 and COV of that with one at TSUKIDATE and HITACHI in The 2011 off the Pacific coast of Tohoku Earthquake.

CONCLUSIONS

Using the time-frequency characteristics of the recorded strong earthquake ground motions in the 2011 off the Pacific coast of Tohoku Earthquake and Masuda's method, several groups of synthesized ground motions are generated. The coefficient of variation (COV) of the maximum displacement responses on the bi-linear single degree of freedom system are shown. It is recognized that COV varies with natural period in elastic domain of the system and the type of the time-frequency characteristic of the synthesized ground motions and COV becomes large as input intensity increases. In this study, we can not find out clear difference between COV of the maximum displacement responses due to the synthesized ground motions with the time-frequency characteristic of the recorded ground motion at KASHIWAZAKI in The Niigataken Chuetsu-oki Earthquake in 2007 and COV of that with one at TSUKIDATE and HITACHI in The 2011 off the Pacific coast of Tohoku Earthquake.

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