TSUNAMI INVERSION ANALYSIS OF
THE GREAT EAST JAPAN EARTHQUAKE

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ABSTRACT: We carried out a tsunami inversion analysis and built a tsunami source model for the Great East Japan Earthquake. We focused attention on the a) tide record, b) flooding height and run-up height, c) crustal movement and d) inundated area. The proposed source model with a magnitude (Mw) of 9.1 consists of mutually independent 80 blocks. The maximum fault slip amount which is located near the trench axis is 56.7m. We confirmed that the observed tsunami data, especially the four parameters described above, were reproduced quite accurately via the model in a well balanced manner.

Key Words: The Great East Japan earthquake, tsunami inversion analysis, tide record, tsunami height, crustal movement, inundated area, tsunami source model

INTRODUCTION

The tsunami caused by the Great East Japan Earthquake on March 11, 2011, struck the Pacific coast of Hokkaido, Tohoku and Kanto regions. This great earthquake and giant tsunami invited heavy losses in the northeastern region of Honshu Island.

This event occurred on the boundary between the Pacific and the Eurasian plates. It was thought that the rupture area extended widely from the region off the coast from Iwate to Ibaraki prefectures. The focal mechanism was estimated as a reverse fault with a compression axis in a WNW-ESE direction. The Headquarters for Earthquake Research Promotion (2002) and Japan Society of Civil Engineers (2002) had evaluated the earthquake and tsunami for mutually individual regions such as the offing of the Miyagi prefecture, the offing of the south Sanriku along the trench and the offing of Fukushima prefecture. According to the evaluation, an occurrence of the simultaneous movement of all these regions was beyond the assumption.

This huge tsunami was recorded at numerous points in many parameters, such as the tide record, flooding height, run-up height, crustal movement, inundated area etc. For the purpose of investigating the mechanism of the earthquake and tsunami, we conducted an analysis on the basis of those records.

As for tsunami source model, some inversion models have been proposed for the purpose of reproducing the tide record, such as Fujii et al. (2011), Tanioka (2011). However, there was no model that could be used to reproduce the tide record, flooding height, run-up height, crustal movement and inundated area in a well-balanced manner. We carried out a tsunami inversion analysis with the aim of building a well-balanced tsunami source model and reproducing an observed tsunami via numerical
SURVEY RESULTS

The target parameters of our inversion were the a) tide record, b) flooding height and run-up height, c) crustal movement and d) inundated area over a broad area (Hokkaido – Chiba Pref.). At first we collected recorded data concerning those four parameters.

We used the data measured by the Japan Meteorological Agency (2011), Ports and Harbors Bureau, Ministry of Land Infrastructure, Transport and Tourism (2011), National Oceanic and Atmospheric Administration (2011) and Earthquake Research Institute, University of Tokyo (2011) etc. as to the observed tide record. The digital data with time intervals of 10 seconds was created for each point using the published tide record. The analog data was digitalized from paper.

As for the observed flooding height and run-up height, we used the results by The 2011 Tohoku Earthquake Tsunami Joint Survey Group (2011) from Hokkaido to Chiba Pref. and by our trace survey in Fukushima Pref. The prerelease version by the Joint Survey Group was used for this study. Among all 3,256 data points by the Joint Survey Group, the following data was excluded and we used a total of 2,820 points.

- Data without location information or flooding height and run-up height
- Data whose degree of confidence is D or W (small tsunami)
- Data with the same latitude, longitude and wave height as those of other data
- Data whose degree of confidence of the elevation is low (data with the description of “using Google map”)
- Data with description “due to local geography or splashes”
- Data with descriptions such as “a larger tsunami hit” etc.
- Data whose trace mark is widely displaced from the coastline to the seaside or landside
- Data with negative values and data of 10cm or less
- Data having the discrepancy with the content in the comments column regarding the flooding height and run-up height
- Data whose distance from the coastline exceeds 1km and the height is less than 1m

As for the crustal movement, we used the results measured by the Geospatial Information Authority of Japan (2011a) for land areas and the Japan Coast Guard (2011) for marine areas.

As for the inundated area, the aerial photos and satellite images taken after the earthquake were used to decipher inundated areas. We divided the area into 73 regions. And we used the results by The Geospatial Information Authority of Japan (2011b) from Aomori to Chiba Pref. and by us in Fukushima Pref.

TSUNAMI INVERSION

In order to study the tsunami caused by the Great East Japan Earthquake, we carried out a tsunami inversion analysis with the aim of building a well-balanced tsunami source model using observed records. We focused attention on the tide record, flooding height, run-up height, crustal movement and inundated area.

Method of Tsunami Inversion Analysis

We used the tsunami inversion method proposed by Annaka et al. (1999). In our inversion, we extended Annaka et al. (1999) for including the data of crustal deformation and the inundated area. Essentially the nonlinear inversion is needed for the inversion based on a nonlinear tsunami analysis. However, it is difficult to put the nonlinear inversion into practice because of the vast computational
Instead of the nonlinear inversion, the linear inversion using the Green function based on the linear tsunami analysis was used by converting the data in the nonlinear space (real space) to the data in the linear space. The conversion was conducted based on the relationship of the calculated values between the nonlinear tsunami analysis and the linear tsunami analysis. In the linear inversion, the following function $J$, the error sums of squares, is minimized.

$$\sum_{i}^{n_i} \{ \log_{10}(A_{\text{Conv}}^i) - \log_{10}(A_{\text{L}}^i) \}^2 + C_2 \sum_{i}^{n_i} \sum_{j}^{n_j} \left\{ \frac{\left[Z_{\text{NL}}^o(t_j)\right]^j}{L_{\text{NL}}^o} - \left[Z_{\text{NL}}^l(t_j)\right]^j \right\}^2$$

$$+ C_3 \sum_{i}^{n_i} \sum_{j}^{3} \left\{B^o_{\text{L},ij} - (B^l_{\text{L},ij})^j \right\}^2 + C_4 \sum_{i}^{n_i} \left\{ \log_{10}(I_{\text{Conv}}^o) - \log_{10}(I_{\text{L}}^o) \right\}^2$$

(1)

where $n_i$: number of data, $C_i$: weighting factor, $A_{\text{Conv}}^O$: converted tsunami height from observed value, $A_L^O$: maximum value of calculated tsunami wave, $Z_{\text{NL}}^O(t_j)$: amplitude of tide record at time $t_j$, $Z_{\text{NL}}^L(t_j)$: amplitude of calculated tsunami wave at time $t_j$, $B^O$: observed value of crustal deformation in $j$-th direction, $B^L$: calculated value of crustal deformation in $j$-th direction, $I_{\text{Conv}}^O$: converted value from observed inundation area, $I_{\text{L}}^O$: calculated value for representing inundation area. Calculated tsunami waves are obtained by the summation of Green functions from the fault slip distribution. The weighting factors are normally determined so that the error sums of squares for the four kinds of data become nearly equal.

The nonlinear tsunami analyses were conducted using the fault slip distribution obtained by the linear inversion and the fitness between the observed and calculated data was evaluated. When the fitness was insufficient, the converted values used in the linear inversion were revised based on the added nonlinear tsunami analyses. Iterations of linear inversion and nonlinear tsunami analysis were continued until the converted values become unchanged and the fitness was sufficient.

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The conditions for the linear tsunami analysis are as follows.

- Basis equation: Linear long wave equation
- Computational scheme: Staggered grid in leap-frog method
- Initial displacement: Method by Mansinha and Smylie (1971)
- Calculated grid division: Shown in Fig. 1. The basic grid interval was set to 1350m, and the grid interval was decreased to 450m, 150m, and 50m as land approaches.

For a nonlinear tsunami analysis, the nonlinear long wave equation was used and a run-up calculation was conducted taking the land topography into account.

**Tsunami Source Model**

The tsunami source model consists of the aggregate of small fault blocks. This is because the shape of the plate boundary surface could be described correctly. A slip angle of a small fault block was defined for each small fault block based on the direction of the relative plate motion, i.e. $N65^\circ W$. We defined the 1,255 small fault elements and arranged them into 80 blocks. The slip amount was set constant in each block in this paper. And we determined the slip amount at each block using the tsunami inversion analysis. Here, we assumed that the rupture start time and duration time is constant for all blocks in this inversion model.

As a result of the inversion analysis we got the fault slip distribution shown in Fig. 2 as the tsunami source model. The moment magnitude (Mw) of this model is 9.1. The fault slip amount is particularly large near the trench axis (maximum slip amount of 56.7m), which shows the same tendency as the model by Fujii et al. (2011) based on the tide record. We presumed that this is because a large inter-plate earthquake near the Japan trench (tsunami earthquake type) occurred at the same time as the inter-plate earthquake off the coast of Miyagi Prefecture and Fukushima Prefecture, and that resulted in the occurrence of large slips along the ocean trench.
Fig. 1 Calculation grid division
(1350m in aqua, 450m in green, 150m in yellow, and 50m in red)

Fig. 2 The fault slip distribution; 80 blocks
Finally, we carried out numerical simulations, a nonlinear forward analysis, using the proposed tsunami source model shown in Fig.2. A comparison of the observed and the analyzed tsunami is as follows.

A comparison of the flooding height and run-up height is shown in Fig.3. In comparison with the 2,820 data points, this model reproduces the tsunami height in \( K=1.04 \) and \( \kappa=1.40 \); reproduction index by Aida (1978). These values satisfy the criteria established by the Japan Society of Civil Engineers (2002), that is \( 0.95<K<1.05 \) and \( \kappa<1.45 \). There are some regions where there is a difference, but in general, the flooding height and run-up height are reproduced quite accurately via our inversion model.

A comparison with the tide record is shown in Fig.4. The overall reproducibility is good, and in particular, the reproducibility of the waveforms of Kushiro and Iwate(M) are good.

The comparisons of the crustal movement observed in the land areas are shown in Fig.5 and that in the marine areas are shown in Fig.6. The reproducibility of the GPS measurement in the land areas is good. In the horizontal direction in the marine areas, the reproducibility is good in general, and the direction is reproduced quite well. In the vertical direction, offshore Miyagi 1, which has the maximum displacement, is reproduced well, however there is one point where direction is reversed.

A comparison of the inundated area is shown in Fig.7. The inundated area from Aomori Prefecture to Chiba Prefecture is reproduced well.

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**Fig.3 Comparison of the flooding height and run-up height**

\[
K_i = \frac{R_i}{H_i} \\
\log K = \frac{1}{n} \sum_{i=1}^{n} \log K_i \\
\log \kappa = \left[ \frac{1}{n} \sum_{i=1}^{n} (\log K_i)^2 - n(\log K)^2 \right]^{1/2}
\]

\( n \) : Number of the locations

\( R_i \) : Observed tsunami height at location \( i \)

\( H_i \) : Calculated tsunami height at location \( i \)

The criteria established by JSCE (2002)

\( 0.95<K<1.05 \) and \( \kappa<1.45 \)
Fig.4 Comparison of the tide record
(a) Crustal movement in horizontal (Left: Calculated, Right: Observed)

(b) Crustal movement in vertical; Subsidence (Left: Calculated, Right: Observed)

Fig. 5 Comparison of the crustal movement in the land area
Fig. 6 Comparison of the crustal movement in the marine area

(a) Crustal movement in horizontal (Red: Calculated, Blue: Observed)

(b) Crustal movement in vertical (Red: Calculated, Blue: Observed)
CONCLUSIONS

For the purpose of investigating the mechanisms of the earthquake and tsunami of the Great East Japan Earthquake on March 11, 2011, we carried out a tsunami inversion analysis. We focused attention on the a) tide record, b) flooding height and run-up height, c) crustal movement and d) inundated area over a wide area (from Hokkaido to Chiba). The proposed source model with a magnitude (Mw) of 9.1 consists of mutually independent 80 blocks, and each block has a different fault slip amount. The maximum fault slip amount which is located near the trench axis is 56.7m. As a result, we confirmed that the observed tsunami data, especially the four parameters described above, are reproduced quite accurately via the model in a well balanced manner.

We assumed that the rupture start time and the duration time are constant for all regions in this inversion model. However these two parameters could be effective in improving the tsunami source model. We think that the investigation of the effect of these parameters is a future task. In addition to the aforementioned parameters, the splay fault and landslide induced by the earthquake could also be effective in improving the model.

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